

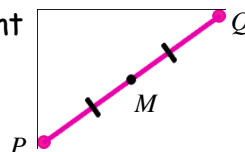
# Grade 10

## ANALYTIC GEOMETRY: MIDPOINT OF A LINE SEGMENT

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Play the **Midpoint Trivia** <http://www.quia.com/cb/167644.html> first.  
You may also go to [www.wiredmath.ca](http://www.wiredmath.ca) for the link.

A **midpoint** is the point on a line segment that divides it into two equal parts.  
 $M$  is the midpoint of  $P$  and  $Q$ .



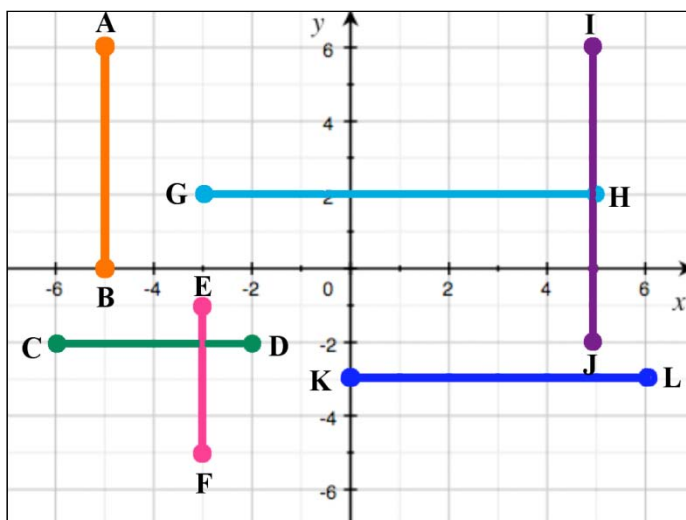
1. a. Find the coordinates of the midpoint of each line segment:

i.  $AB$  \_\_\_\_\_ ii.  $CD$  \_\_\_\_\_

iii.  $EF$  \_\_\_\_\_ iv.  $GH$  \_\_\_\_\_

v.  $IJ$  \_\_\_\_\_ vi.  $KL$  \_\_\_\_\_

- b. What is the relationship between the the coordinates of the midpoint of the line segment and the coordinates of the endpoints of the horizontal and vertical line segments?




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- c. Use the result you found in part b. to determine the coordinates of the midpoint of each of the following line segments:

i.  $M(1,2)$  and  $N(5,2)$       ii.  $O(-4,-7)$  and  $P(-4,-1)$       iii.  $Q(-5,3)$  and  $R(-1,3)$

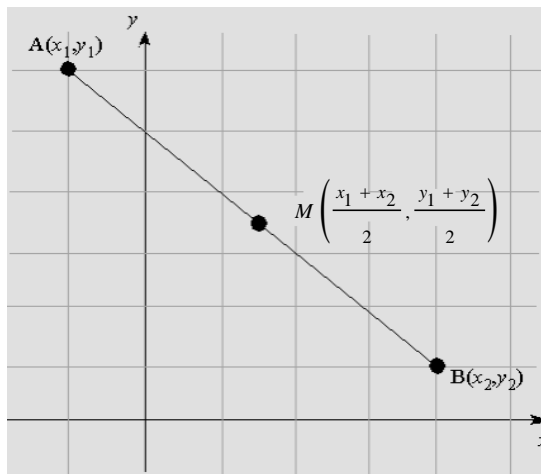
iv.  $S(0,4)$  and  $T(0,9)$       v.  $U(-2,0)$  and  $V(6,0)$       vi.  $W(2,7)$  and  $X(2,-3)$

vii.  $Y(3,1)$  and  $Z(3,-9)$       viii.  $A(6,3)$  and  $B(-1,3)$       ix.  $C(-19,-41)$  and  $D(-19,-2)$

## Midpoint Formula

The coordinates of the midpoint,  $M$ , of a line segment with endpoints  $A(x_1, y_1)$  and  $B(x_2, y_2)$  are given by

$$\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right).$$



- Determine the coordinates of the midpoint of each line segment with the given endpoints.
  - (4,5) and (2,9)
  - (-8,3) and (2,7)
  - (41,105) and (29,45)
  - (1.4,3.1) and (4.4,0.3)
  - $\left(\frac{1}{2}, \frac{3}{2}\right)$  and  $\left(\frac{7}{2}, \frac{1}{2}\right)$
  - ( $a, b$ ) and ( $c, d$ )
- The coordinates of the vertices of a rectangle are  $A(-1, 2)$ ,  $B(-1, 4)$ ,  $C(5, 4)$ , and  $D(5, 2)$ . Verify that the diagonals bisect each other.
- A diameter of a circle joins the points  $A(-3, 7)$  and  $B(-9, 15)$ .
  - Find the centre of the circle.
  - What is the length of the radius?
- A line segment  $AB$  has an endpoint  $A(1, 2)$  and the midpoint is  $M(4, 3)$ . What are the coordinates of the endpoint  $B$ ?
  - A line segment  $XY$  has an endpoint  $X(-3, 0)$  and the midpoint is  $M(-8, 5)$ . What are the coordinates of the endpoint  $Y$ ?
  - A line segment  $EF$  has endpoints  $E(5, -1)$  and  $F(-3, a)$ . The midpoint has coordinates  $M(b, 4)$ . What are the values of  $a$  and  $b$ ?
- A right triangle has vertices  $A(-1, -2)$ ,  $B(-1, 4)$  and  $C(5, -2)$  and  $\angle A = 90^\circ$ . Verify that the midpoint of the hypotenuse is equidistant from each vertex of the triangle.
- The line segment  $PQ$  has an endpoint  $P(2, 8)$ . The midpoint,  $M$ , of  $PQ$  is on the  $y$ -axis and the other endpoint  $Q$  is on the  $x$ -axis. Find the coordinates of  $M$  and  $Q$ .

Don't forget to try this math drill now! Go to [www.wiredmath.ca](http://www.wiredmath.ca) for the links.

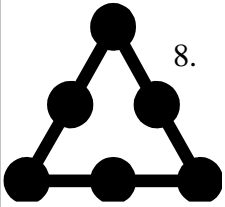


**TRY THIS!**

**Quiz: Coordinate Geometry**

<http://library.thinkquest.org/20991/quizzes/geo/q11/test.html>

## CHALLENGE YOURSELF!



8. The three midpoints of the sides of triangle  $ABC$  are  $E(-2,3)$ ,  $F(2,1)$  and  $G(-1,-3)$ . What are the coordinates of the vertices of the triangle?
9. The centroid of a triangle is the point of intersection of the medians. Triangle  $PQR$  has vertices  $P(-6,2)$ ,  $Q(8,-2)$  and  $R(4,6)$ .
- What are the coordinates of the centroid?
  - Verify that the ratios  $PG$  to  $GN$  and  $RG$  to  $GM$  are equal, where  $G$  is the centroid.
  - Determine the ratio  $QG$  to  $GO$ , where  $G$  is the centroid.



### Did You Know?

Performers on the high wire must grip the long pole that they carry for stability at its midpoint.

10.
  - Given the two points  $P(a,b)$  and  $Q(b,a)$ , prove that the midpoint of  $PQ$  lies on the line  $y = x$ .
  - Prove that  $PQ$  is perpendicular to the line with the equation  $y = x$ .
  - Explain the significance of these results in terms of a function  $f$  and its inverse  $f^{-1}$ .
11. If  $(2,5)$  is the midpoint of the line segment joining  $A(5,y)$  and  $B(x,7)$ , then determine the value of  $x + y$ .

## EXTENSIONS

12. The endpoints of a diameter of a circle are  $S(-12,-9)$  and  $T(12,9)$ .
- Find the coordinates of the centre of the circle.
  - Find the radius of the circle.
  - Find the circumference and area of the circle.
  - Find the equation of the circle.
13. Repeat 12. using  $U(-5,2)$  and  $V(11,-10)$ .