

Grade 7

GEOMETRY AND SPATIAL SENSE:

CARTESIAN COORDINATE SYSTEM AND DILATATIONS

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Play the **Coordinate Plane Game** first!

Click on <http://www.math-play.com/Coordinate Plane Game/Coordinate Plane Game.html>

Or you may go to <http://www.wiredmath.ca> for the link.

A **Cartesian coordinate plane** has two intersecting number lines that form axes.

The horizontal axis is called the **x -axis** and the vertical axis is called the **y -axis**. The axes intersect at the point called the **origin**.

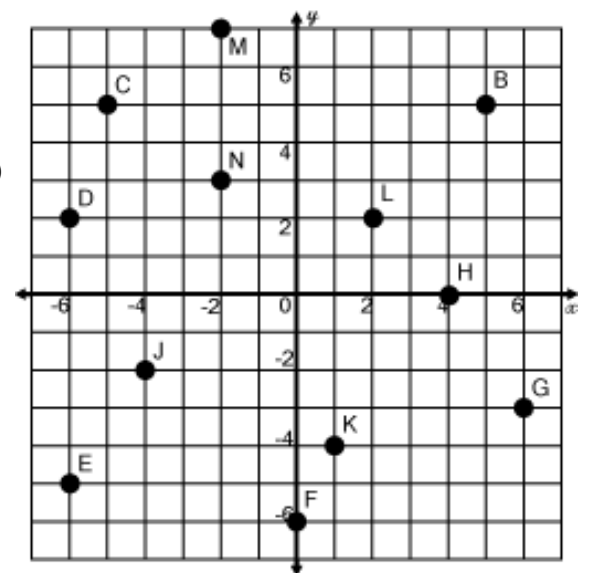
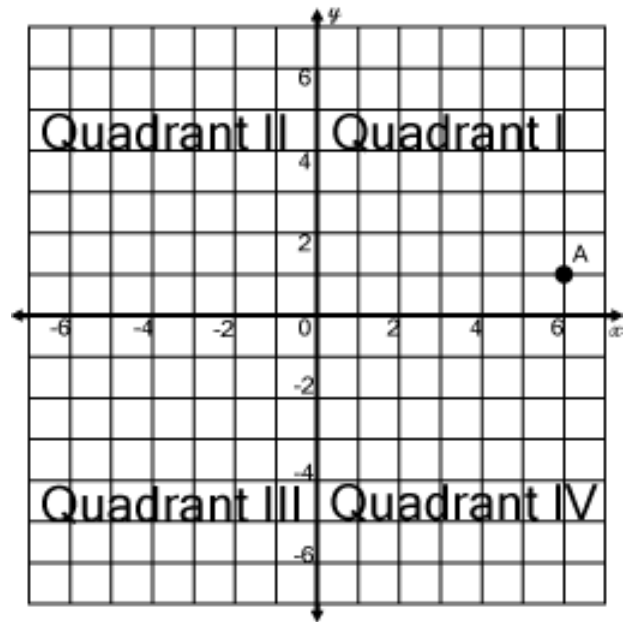
The axes divide the coordinate plane into four **quadrants**.

A point on the plane can be described by its x and y coordinates. These coordinates are written as an ordered pair: (x, y) .

- The coordinates of the origin are $(0, 0)$
- The coordinates of point A are $(6, 1)$

To plot $(6, 1)$ on the plane, start at the origin and move 6 units to the right and 1 unit up.

Always start with the x -coordinate.

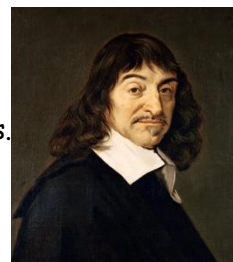


- Name the point that has the coordinates.
a. $(2, 2)$ b. $(-6, 2)$ c. $(1, -4)$ d. $(0, -6)$ e. $(-4, -2)$
- Write the coordinates of each point.
a. B b. G c. E d. N e. H
- In what quadrant is each point located?
a. C b. J c. L d. M e. K

4. On grid paper, make four Cartesian planes by drawing four sets of axes. Plot each set of points on a different plane and join them in order to form a quadrilateral. Identify the quadrilateral.
- $A(1, 1)$, $B(1, 5)$, $C(-3, 5)$, $D(-3, 1)$
 - $J(1, -3)$, $K(5, 1)$, $L(8, 1)$, $M(4, -3)$
 - $P(-3, 0)$, $Q(-6, -2)$, $R(5, -2)$, $S(2, 0)$
 - $W(-1, 1)$, $X(0, 3)$, $Y(4, 1)$, $Z(3, -1)$

A Slice of History

Cartesian means relating to the French mathematician and philosopher René Descartes. He developed the coordinate plane system in 1637.



http://en.wikipedia.org/wiki/Cartesian_coordinate_system



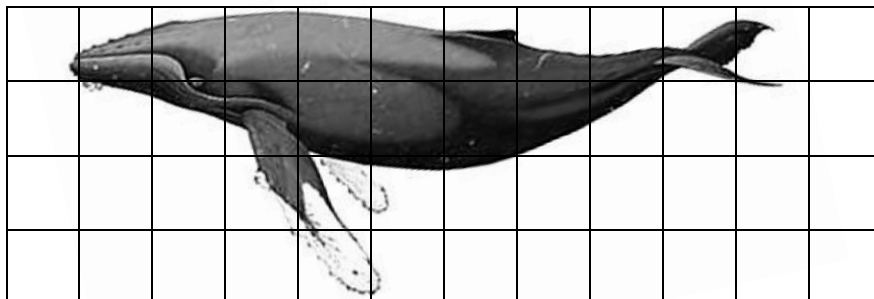
Try this math drill now!
Go to www.wiredmath.ca for the link or use the link below!



Draw a Tyrannosaurus-Rex by plotting points.

http://www.mathsisfun.com/t_rex.html

- 5.
- Draw a grid of squares with sides twice as long as in the grid below.
 - Make an enlargement of the whale by copying the contents of each square in the grid below into corresponding squares in the grid you made in part a.
 - Make a reduction of the whale by using a grid of squares with sides half as long as in the grid below.



A **dilatation** is a transformation that changes the size of an object.

If the image is larger than the original, the dilatation is an **enlargement**.

If the lengths have been doubled, the scale factor is 2.

If the image is smaller than the original, the dilatation is a **reduction**.

If the lengths have been halved, the scale factor is $\frac{1}{2}$.

- 6.
- Draw a rectangle that has vertices $A(2, 4)$, $B(2, 2)$, $C(3, 2)$ and $D(3, 4)$ on a Cartesian plane.
 - Multiply the x and y coordinate of each ordered pair by 2. So the image of the point $A(2, 4)$ is $A'(4, 8)$. Plot the rectangle that has those vertices on the same grid.
 - Draw a line from the origin, O , through A . Does the line pass through A' ?
 - Does the line from the origin that passes through B pass through B' ? C and C' ? D and D' ?
 - Using a ruler, compare the lengths of OA and OA' . Do the same for OB and OB' , OC and OC' , and OD and OD' .

Expectations: i) describe location in the four quadrants of a coordinate system, dilate two-dimensional shapes, and apply transformations to create and analyse designs; ii) plot points using all four quadrants of a Cartesian coordinate plane; iii) identify, perform, and describe dilations through investigation using a variety of tools. For more activities and resources from the University of Waterloo's Faculty of Mathematics, please visit www.cemc.uwaterloo.ca.

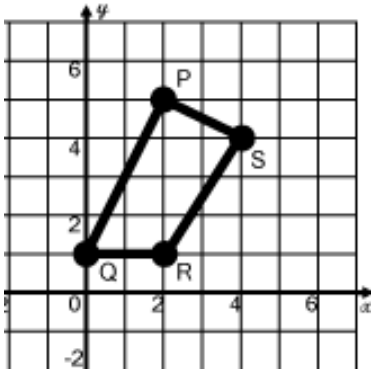
When a figure is plotted on a coordinate grid, it can be dilated by

- i) multiplying the coordinates of the vertices by a scale factor
- ii) drawing a line segment from the origin through the vertex and multiplying its length by a scale factor

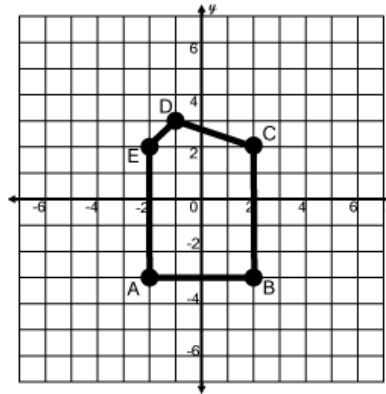
The **dilatation centre** is usually the origin, (0, 0)

7. Enlarge the figure by a scale factor of 3, with the dilatation centre (0, 0). Write the coordinates of the vertices of the image. (Hint: Find the coordinates of the image points first, and use them to draw the enlarged figure.)

a.

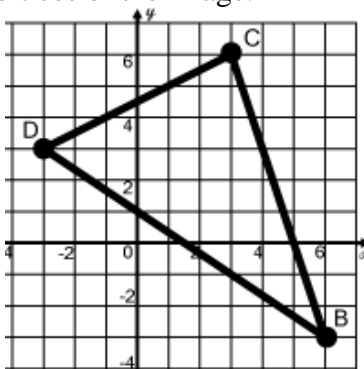


b.

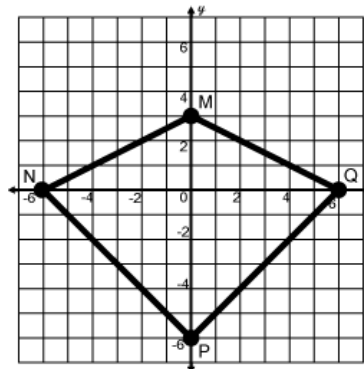


8. Reduce the figure by a scale factor of $\frac{1}{3}$, with the dilatation centre (0, 0). Write the coordinates of the vertices of the image.

a.



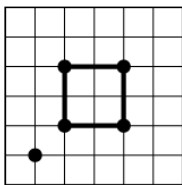
b.



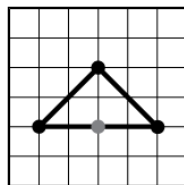
The letter O is usually reserved for the point (0, 0), the origin.

9. Copy each figure onto grid paper. Draw the dilatation image, given the dilatation centre and the scale factor.

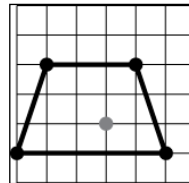
a. Scale factor 3



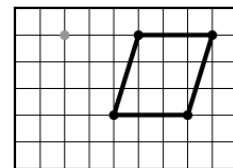
b. Scale factor 2



c. Scale factor 2



d. Scale factor $\frac{1}{2}$



10. a. A photographer printed a digital photograph that was 1625 pixels by 1300 pixels into a photograph of dimensions 25 cm by 20 cm. What was the scale factor?
 b. Suppose the photographer wanted to double the dimensions of the 25 cm by 20 cm photograph. What would the new dimensions be? What would the scale factor be compared to the original digital photograph?

Research

Are maps examples of dilatations? Explain.
Investigate 3 different maps. Find the scale factor of each map.
What is another name for the scale factor of a map?

CHALLENGE YOURSELF!

11. Given the coordinates of three vertices of a rectangle, use a grid to find the coordinates of the fourth vertex.
 - a. $L(-9, -7)$, $M(-9, 1)$, $N(5, -7)$
 - b. $P(-3, 1)$, $Q(0, 4)$, $R(5, -1)$
12. Plot the points on a Cartesian plane and join them in order. Find the area of the figure created.
 - a. $A(-3, 2)$, $B(1, 2)$, $C(1, 5)$, $D(0, 9)$, $E(-3, 5)$
 - b. $F(-1, 0)$, $G(2, 0)$, $H(2, -3)$, $I(-1, -3)$, $J(-3, -1)$, $K(-1, -1)$
13.
 - a. Dilate each figure in question 11 by a scale factor of 3. State the coordinates of the vertices of the new figures and draw them on a Cartesian plane.
 - b. Find the area of each dilated image.
 - c. How does the area of the dilated image compare to the area of the original image? Refer to the scale factor in your answer.

EXTENSION!

14. Suppose you and a friend are outside on a sunny day. You decide to calculate the height of a tall tree. You measure the length of the shadow that the tree casts and find that it is 10 m. Then you measure the length of the shadow that your friend casts and find that it is 1 m. You know that your friend is 150 cm tall. Determine the height of the tree.
15. The volume of a cube is 64 cm^3 .
 - a. If the cube is enlarged so its volume is 1000 cm^3 , what is the scale factor?
 - b. If the cube is reduced so its volume is 8 cm^3 , what is the scale factor?

Battleship!

Battleship is a game played by two people. It involves guessing the position of your opponent's battleships on a grid. Go to <http://www.funorama.com/files/battleships.pdf> to download a classic version of the game.
Now create and play your own version of Battleship using a Cartesian-style grid!