Grade 7

Geometry and Spatial Sense: Transformations

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Play the **Transformations Game** first!

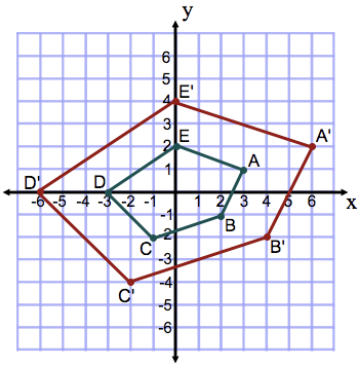
Click *on* http://www.bbc.co.uk/schools/ks2bitesize/maths/activities/transformation.shtml

Or you may go to www.wiredmath.ca for the link.

A **dilatation** is a transformation where the size of an object changes. An enlargement is when the image is larger than the original. A reduction is when the image is smaller than the original.

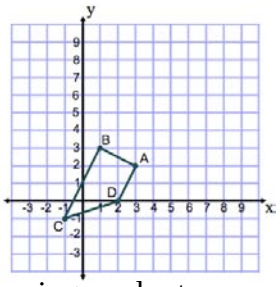
When a figure is plotted on a coordinate grid, it can be dilated by multiplying the coordinates of the vertices by a scale factor.

E.g. On the right, the figure is dilated by a scale factor of 2.

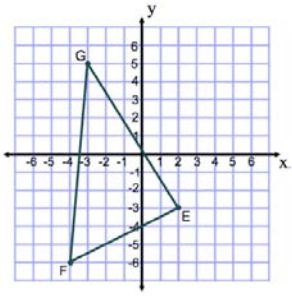


1. Dilate the figure by the given scale factor.

a. Scale factor 3



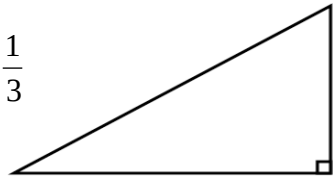
b. Scale factor $\frac{1}{2}$



2. Dilate the figure by using a ruler to measure the length of each side and multiplying the length of each side by the given scale factor.

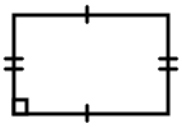
a.

Scale factor $\frac{1}{3}$



b.

Scale factor 4



A **translation** is a transformation where the location of an object changes. Another name for a translation is a slide because the figure slides along a straight line.

When a figure is plotted on a coordinate grid, a translation can be described by

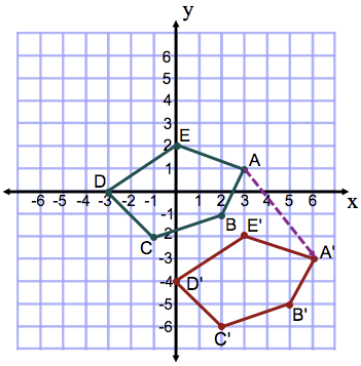
i) an ordered pair (horizontal movement, vertical movement)

E.g. The ordered pair $(3, -4)$ describes the translation at the right.

ii) a mapping $(x, y) \rightarrow (x + \text{horizontal movement}, y + \text{vertical movement})$

E.g. The mapping at the right is $(x, y) \rightarrow (x + 3, y - 4)$

So, $A(3,1) \rightarrow A'(6,-3)$, $B(2,-1) \rightarrow B'(5,-5)$, and so on.

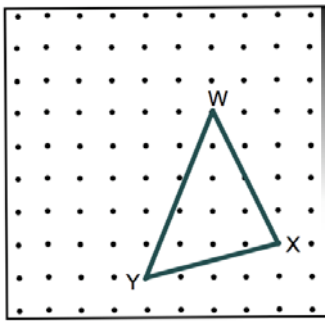


Expectations: i) describe location in the four quadrants of a coordinate system, dilate two-dimensional shapes, and apply transformations to create and analyse designs; ii) create and analyse designs involving translations, reflections, dilatations, and/or simple rotations of two-dimensional shapes, using a variety of tools.

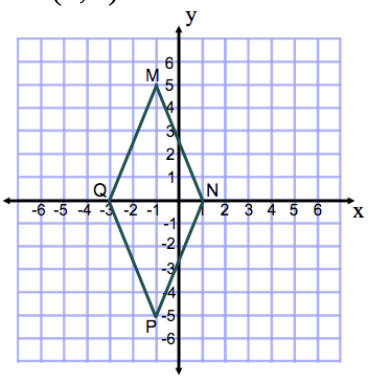
For more activities and resources from the University of Waterloo's Faculty of Mathematics, please visit www.cemc.uwaterloo.ca.

3. Translate the figure.

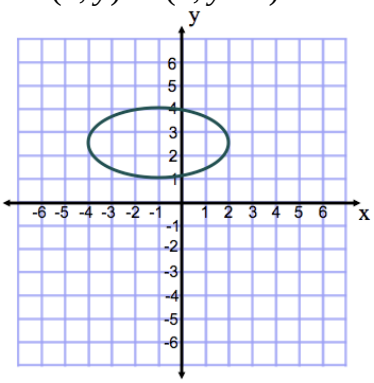
a. 3 units left, 2 units up



b. $(4, 0)$



c. $(x, y) \rightarrow (x, y-5)$



4. A figure is translated $(-2, 6)$. What translation would move the translated image back to its original position?

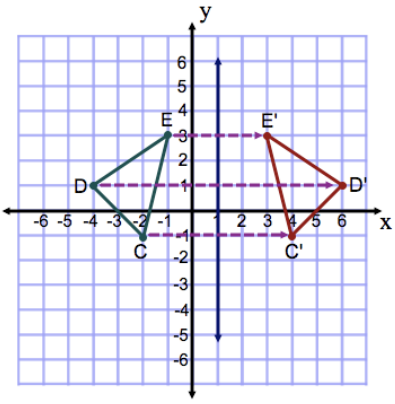
5. You are standing at the bottom of a flight of 12 stairs. Each step is 30 cm deep and 30 cm high.
   a. Write a translation in the form (x, y) to represent your trip from the bottom to the top.
   b. Write a translation as a mapping to represent a trip from the top of the stairs to the bottom.

A **reflection** is a transformation that can be described as a flip about a **mirror line** or a **reflection line**. This is also referred to as **line symmetry**.

On a grid, a figure and its reflected image are the same perpendicular distance away from the reflection line.

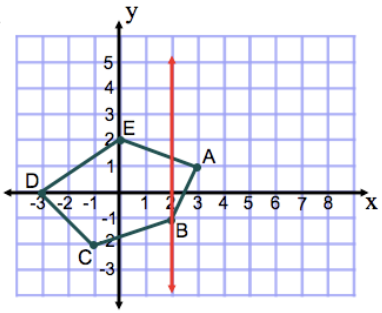
E.g. Each image point at the right is the same distance from the vertical reflection line as the corresponding point from the original image. This reflection may be written as a mapping.

$C(-2,-1) \rightarrow C'(4,-1)$, $D(-4,1) \rightarrow D'(6,1)$, $E(-1,3) \rightarrow E'(3,3)$

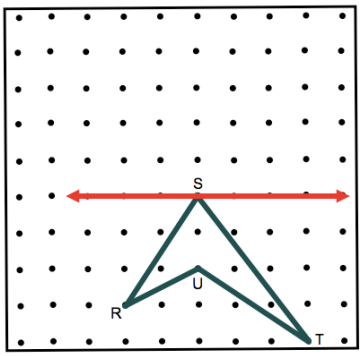


6. Draw the image of the figure around the given reflection line.
(Hint: Find the image of each vertex and then connect them.)

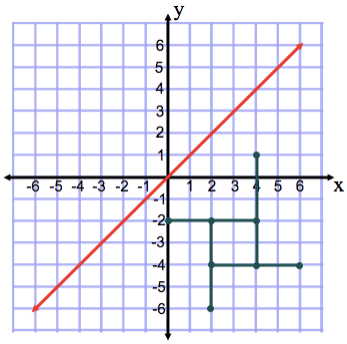
a.



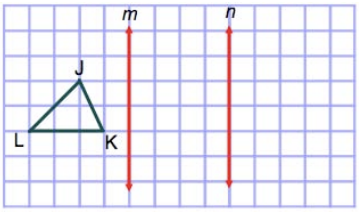
b.



c.



7. a. Reflect ΔPJK in the reflection line m to give $\Delta J'K'L'$.
   Then reflect $\Delta J'K'L'$ in line n to give $\Delta J''K''L''$.
   b. What single translation would map ΔJKL onto $\Delta J''K''L''$?



Expectations: i) describe location in the four quadrants of a coordinate system, dilate two-dimensional shapes, and apply transformations to create and analyse designs; ii) create and analyse designs involving translations, reflections, dilatations, and/or simple rotations of two-dimensional shapes, using a variety of tools.

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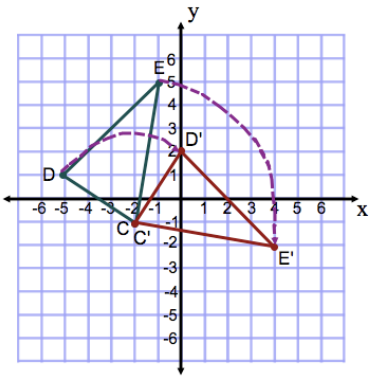
A **rotation** is a transformation that can be described as a turn about a point. This point is called the **turn centre**.

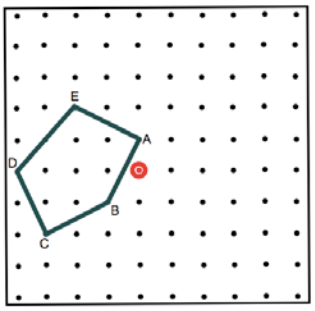
E.g. $\Delta C'D'E'$ is the rotation image of ΔCDE. The turn centre is $C(-2,-1)$.
The rotation is a $\frac{1}{4}$ turn clockwise or 270°.
Note: The image $\Delta C'D'E'$ is the same if ΔCDE is rotated a $\frac{3}{4}$ turn counter clockwise.

90° = $\frac{1}{4}$ turn counter clockwise 180° = $\frac{1}{2}$ turn
270° = $\frac{3}{4}$ turn counter clockwise 360° = 1 turn

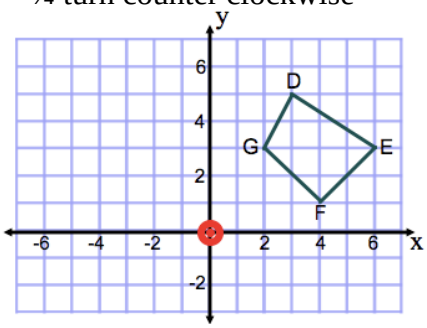


8. Draw the image after a rotation about the turn centre. (Hint: Trace the image using tracing paper, rotate the tracing paper as required, mark the image point of each vertex and then connect them to make the rotation image.)

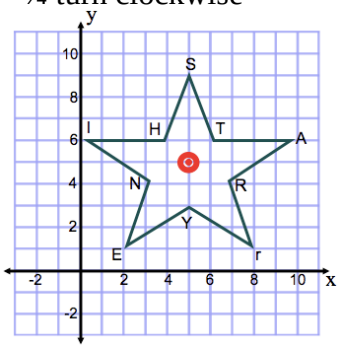
a. ½ turn clockwise



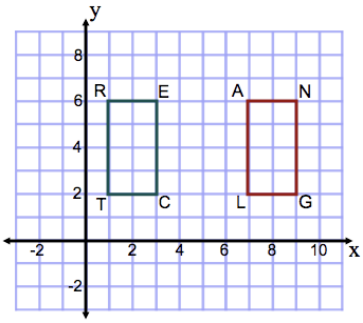
b. ¼ turn counter clockwise



c. ¼ turn clockwise

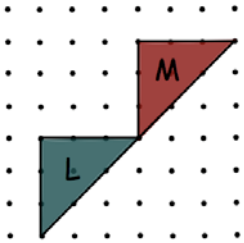


9.



a. Is rectangle ANGL a translation image of rectangle RECT? If so, explain the transformation in words.
b. Is rectangle ANGL a reflection image of rectangle RECT? If so, where is the reflection line?
c. Is rectangle ANGL a rotation image of rectangle RECT? If so, what are the coordinates of the turn centre? What is the mapping of each vertex?

10. Describe Figure L as a transformation image of Figure M in two different ways.



Watch this video about Transformations!
Go to www.wiredmath.ca or click on the link below.

Real Life Transformations Video
www.blackgold.ab.ca/ict/divison3/exploringtessellations/images/transformations.mov

Think of some other examples where dilatations, translations, reflections, and rotations are seen in everyday life.

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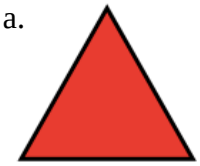
11. This is a pattern for a brick walkway. Use transformations to describe how to construct this design.

12. Use the figure and transformations to create a design on dot paper or grid paper.

a. 

b.

c.

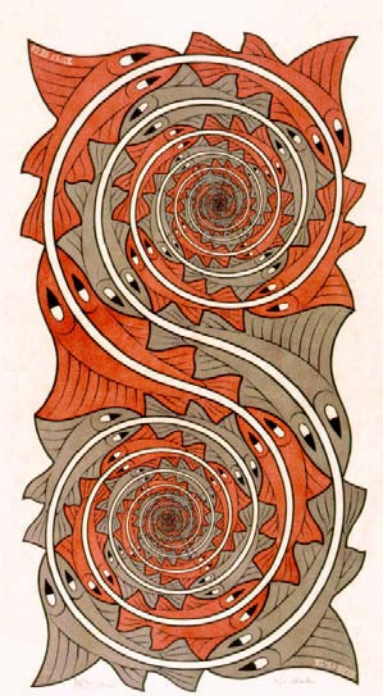
A design is called a **tessellation** if the plane is covered by repeated use of one shape, without gaps or overlaps.

Which of your designs from number 12 were tessellations?

Play the Tessellations and Transformations Game now!
Click on www.learnalberta.ca/content/mejhm/html/object_interactives/transformations/use_it.html
or go to www.wiredmath.ca for the link.

13. Use *Geometer's Sketchpad* to make your own designs using geometric figures and transformations. Instructions for using *Geometer's Sketchpad* are on the following page.

14. M.C. Escher (1898 – 1972) became a very famous graphic artist. Three of his pieces are shown here. Identify the transformations that Escher used in each piece.



Visit www.mcescher.com to learn more about M.C. Escher and see more of his artwork.

Now create your own Escher-style art by applying transformations!

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Geometer's Sketchpad Instructions

Open *The Geometer's Sketchpad* and choose New Sketch from the File menu.

Making Grid Paper on the Screen

It is a good idea to use grid paper or dot paper when transforming figures so let's make grid paper on the screen. Choose **Preferences** from the Edit menu and make sure that the distance units are in cm.
From the **Graph** menu, choose **Define Coordinate System**.
Click on each axis and the two red dots. From the **Display** menu, choose **Hide Objects**. Now the screen looks like a plain piece of grid paper.
From the **Graph** menu, choose **Snap Points**. This will only allow points to be placed on the crosses of the grid.

Creating a Figure

To create a figure, choose the line tool from the **Toolbox**. Click and drag to construct lines.
Construct a shape to perform transformations on.

Dilatating a Figure

1. Choose the point tool from the **Toolbox** and place a point near your figure. Select the point and choose **Mark Centre** from the **Transform** menu.
2. Choose the arrow from the **Toolbox** and click on each side of your figure to select it.
3. From the **Transform** menu, choose **Dilate**.
4. Experiment with different **Scale Factors**. Click **Dilate** to see your original figure and the Dilated image.
5. If you drag a vertex on your original figure, how does the dilatation image change?

Translating a Figure

1. Choose the arrow from the **Toolbox** and click on each side of your figure to select it.
2. From the **Transform** menu, choose **Translate**.
3. Under **Translation Vector**, choose **Rectangular**.
4. Enter values for the **Horizontal** and **Vertical** translations. (Make sure **Fixed Distance** is selected.)
5. Click **Translate** to see your original figure and the translated image.
6. If you drag a vertex on your original figure, how does the translation image change?

Reflecting a Figure

1. Draw a line close to the figure you want to reflect. Select the line and choose **Mark Mirror** from the **Transform** menu.
2. Choose the arrow from the **Toolbox** and click on each side of your figure to select it.
3. From the **Transform** menu, choose **Reflect**.
4. If you drag a vertex on your original figure, how does the reflection image change?
5. If you drag either end of the mirror line, how does the reflection image change?

Rotating a Figure

1. Choose the point tool from the **Toolbox** and place a point near your figure. Select the point and choose **Mark Centre** from the **Transform** menu.
2. Choose the arrow from the **Toolbox** and click on each side of your figure to select it.
3. From the **Transform** menu, choose **Rotate**.
4. Choose **Fixed Angle**, enter an angle, and click rotate to see your original figure and the rotated image. Experiment with different angles.
5. If you drag a vertex on your original figure, how does the rotation image change?