

# Grade 8

## NUMBER SENSE AND NUMERATION: PRIMES AND PRIME FACTORS

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Play **The Primes Game** [http://hotmath.com/hotmath\\_help/games/numbercop/numbercop\\_hotmath.swf](http://hotmath.com/hotmath_help/games/numbercop/numbercop_hotmath.swf) and **The Factors Games** [http://nlvm.usu.edu/en/nav/frames\\_asid\\_202\\_g\\_3\\_t\\_1.html](http://nlvm.usu.edu/en/nav/frames_asid_202_g_3_t_1.html) first!  
You may also go to [www.wiredmath.ca](http://www.wiredmath.ca) for the link.

**Prime number:** A natural number greater than 1 that has exactly two factors, itself and 1. Otherwise, it is called composite.

**Composite number:** A number that has factors in addition to itself and 1. For example,  $6 = 2 \times 3$ .

- Identify each of these numbers as prime, composite, or neither prime nor composite.
  - 100
  - 33
  - 47
  - 11
  - 43
  - 13
  - 1
  - 91
  - 97
  - 81
  - 57
  - 59
- List all the primes less than 200.
- All even numbers are composite numbers. True or false? Give a counterexample if it's false. (Note: Counterexample is an example that refutes or disproves a hypothesis, proposition, or theorem.)



### A Slice of History



An influential master mathematician Leonhard Euler (1707-1783) expressed in 1751:

"Mathematicians have tried in vain to this day to discover some order in the sequence of prime numbers, and we have reason to believe that it is a mystery into which the human mind will never penetrate."

### Did You Know?

- There are infinitely many prime numbers.
- Every integer can be written as a product of primes.

For example,  $360 = 2^3 \times 3^2 \times 5$ .

**Factors:** Natural numbers that divide evenly into a given number. For example, 3 is a factor of 12 because  $4 \times 3 = 12$ .

**Prime factors:** A natural number that is a prime number. For example, 2 and 3 are prime factors of 12, but the other factors of 12 (1, 4, 6 and 12) are not prime factors.

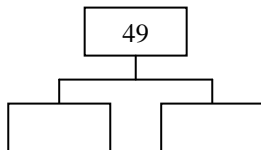
**Prime factorization:** An expression showing a composite number as the product of its prime factors. For example, 42 can be factored as  $2 \times 3 \times 7$ .

4. Of the following, the quotient which is NOT a natural number is  
 a.  $2834 \div 2$     b.  $509 \div 5$     c.  $2100 \div 7$     d.  $350 \div 5$     e.  $380 \div 20$

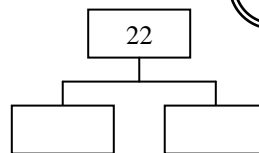
5. a. List all the factors of 60.  
 b. Which of these numbers are prime factors?

6. Complete the following factor trees.

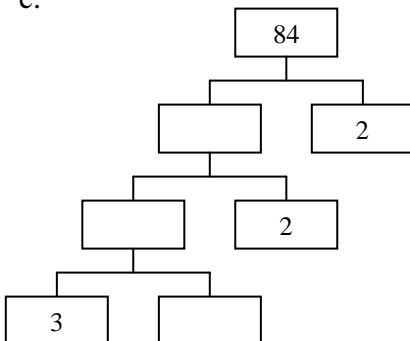
a.



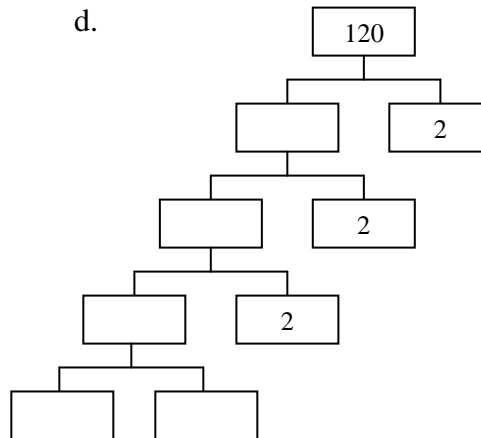
b.



c.



d.



**Interesting Fact!**

1 is neither a composite nor a prime number.

7. a. Besides 1 and itself, can the number 31,500 have factor greater than 15,750? Why or why not?  
 b. Refer to part a, make a general conclusion about any factor (excluding 1 and itself) of an integer  $n$ .

8. Write as the product of prime factors.

- a. 92  
 b. 100  
 c. 400  
 d. 1516  
 e. 567  
 f. 1008

9. The prime factors of a composite number must be less than the square root of the composite number. True or false? Provide a counterexample if false.

**Prime Factorization**

Method: Repeatedly divide by prime numbers

E.g. Prime Factorization of 96 (by division):

$$\begin{array}{r} 2 \overline{)96} \\ 2 \overline{)48} \\ 2 \overline{)24} \\ 2 \overline{)12} \\ 2 \overline{)6} \\ 3 \end{array}$$

Therefore,  $96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3$ .

Prime factorization is used to determine the greatest common factor (GCF) and the least common multiple (LCM).

- **Greatest common factor (GCF):** The largest factor that two or more numbers have in common.
- **Least common multiple (LCM):** The smallest number that two numbers can divide into evenly.

### Determining GCF

E.g. Determine the GCF of 300 and 375.

Method: Write both numbers as the product of prime factors, circle the prime factors that are common to both numbers and then multiply the numbers from each pair together.

$$300 = 2 \times 2 \times \textcircled{3} \times \textcircled{5} \times \textcircled{5}$$

$$375 = \textcircled{3} \times \textcircled{5} \times \textcircled{5} \times 5$$

Therefore,  $3 \times 5 \times 5 = 75$  is the GCF of 300 and 375.

### Determining LCM

E.g. Determine the LCM of 300 and 375.

Method: Write both numbers as the product of prime factors. Circle all the prime factors in the smaller number and then circle any prime factor of the larger number that has not already been circled in the smaller number. Multiply the numbers that are circled.

$$300 = \textcircled{2} \times \textcircled{2} \times \textcircled{3} \times \textcircled{5} \times \textcircled{5}$$

$$375 = 3 \times \textcircled{5} \times 5 \times 5$$

Therefore,  $2 \times 2 \times 3 \times 5 \times 5 \times 5 = 1500$  is the LCM of 300 and 375.

### Did You Know?

Banks use prime numbers to provide security for ATM machines.



10. Determine the GCF and LCM of the following pairs.

a. 25, 90

b. 23, 115

c. 64, 81

d. 96, 168

e. 36, 90

11. Determine the greatest common factor for each group of numbers.

a. 36, 54, 92

b. 51, 66, 39

c. 30, 45, 160

d. 27, 42, 525

12. Can the fraction  $\frac{320}{15309}$  be reduced any further? Why?

13. The bells at two schools ring on different schedules. One bell rings every 45 minutes, the other rings every 60 minutes. If both bells ring at 8:45, what time will it be when both bells ring again at the same time?



14. True or false? Explain.

a. The GCF of two different positive integers can be greater than either number.

b. The LCM of two different positive integers can be smaller than either number.

c. The GCF can be one of the numbers.

Don't forget to try these math drills now! Go to [www.wiredmath.ca](http://www.wiredmath.ca) for the link.

**TRY THESE!**

**Greatest Common Factor**

<http://www.aaamath.com/g72b-grt-com-fac.html>

**Least Common Multiple**

<http://www.aaamath.com/fra66i-lcm.html>



**CHALLENGE YOURSELF!**

15. Determine the sum of the GCF and LCM of the numbers 4, 20 and 28.
16. Other than the number itself, what is the next largest factor of 1001?
17. Find a prime that can be expressed in the form  $n^2 + 2$ .
18. A bulletin board measures 64 cm by 96 cm. Identical square pieces of paper, with side lengths having a positive integer, are to cover the entire bulletin board. Determine the minimum number of square pieces of paper that are needed.
19. You are interviewed to be a programmer of a software company and you are asked to solve the following Math problem:  
*A password is made up with 6 digits (0-9). Your task is to break the password.*  
Hints:
  - a. The first 3 digits can be obtained by finding the total number of positive integers that are less than 500 and are not divisible by 2 or 3.
  - b. The number that you have obtained from part a. should be a prime number.
  - c. The last 3-digits is made up by the largest prime less than 1000.

Password: \_\_\_\_\_



**EXTENSION**

A **perfect number** is a number that equals the sum of its proper divisors.

For example, 6 is perfect since  $6 = 1 + 2 + 3$ .

20. What is the next perfect number after 6?
21. a. How many of the three-digit numbers that can be made from all of the digits 1, 3 and 7 and used only once each are prime?  
b. Repeat using 2, 3 and 4.