The CENTRE for EDUCATION in MATHEMATICS and COMPUTING

WATERLOO MATHEMATICS

Grade 8

Number Sense and Numeration: Ratio, Rates and Percentages

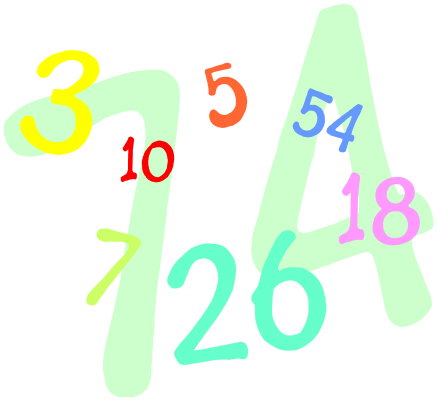
Play the **Balloon Invaders Percentage Game** first.
Click on *http://www.mathplayground.com/balloon_invaders_percent.html* to play.
You may also go to *www.wiredmath.ca* for the link.

Ratio: A comparison of two or more quantities with the same units. For example, the ratio of the number of students who enjoy mathematics to the total number of students in a class is 7 to 8. This is written as 7:8 or $\frac{7}{8}$.

Equivalent Ratios: Multiplying or dividing each term of a ratio by the same non-zero number produces equivalent ratios. For example, 2:10, 3:15 and 1:5 are equivalent ratios. The ratio 1:5 is in lowest terms because its terms have no common factor.

Rate: A comparison of two or more quantities measured in different units. For example, Lisa typed 100 words in 5 minutes. So, the rate is $\frac{100\,\text{words}}{5\,\text{min}} = 20\,\text{words / min}$.

1. Write in lowest terms.
   a. 4:2 = ___:___
   b. 16:12 = ___:___
   c. 25:15 = ___:___
   d. 120:46 = ___:___
   e. 35:15:20 = ___:___:___

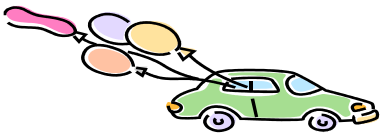


2. Complete each blank.
   a. ___:18 = 6:3
   b. 18:60 = 3: ___
   c. ___:15 = 2:3
   d. 90:96 = ___:16
   e. 72:36:120 = 12: ___:___

3. Write a ratio, in lowest terms, to describe each of the following comparisons.
   a. A class has 15 boys and 12 girls.
   b. In his pencil box, Brian has 4 pencils and 6 pens.
   c. Jane collected 33 Chinese stamps and 22 Canadian stamps.
   d. Soccer team Tiger has played 40 games this season and won 28 of those. What is the ratio of the number of games won to games lost? What is the ratio of the number of games lost to the total number of games?
   e. What is ratio of the area of a circle to the square of its radius?

Expectations: i) demonstrate an understanding of rate as a comparison, or ratio, of two measurements with different units. ii) solve problems involving the calculation of unit rates. *For more activities and resources from the University of Waterloo's Faculty of Mathematics, please visit www.cemc.uwaterloo.ca.*

4. Frank drove 320 km in 4 hours.
   a. What was his average speed?
   b. If he drives an extra 2 hours, how far will he have driven in total?



5. A bike is in fourth gear. When the pedals turn three times, the rear wheel turns seven times.
   a. When the pedals turn 6 times, how many times does the rear wheel turn?
   b. If the wheel's circumference is 220 cm, how far has been travelled when the pedals turn 6 times?

6. Many of the grade 8 students participated in the annual school ski trip. Among the participants, 50 students snowboarded. The ratio of the number of students who skied to the number of students who snowboarded was 3:2.
   a. How many students skied?
   b. What number of grade 8 students participated in the ski trip?

7. The angles of a triangle are in the ratio 3:4:5. What are the measures of each angle?

8. If the length of each side of a square is doubled, what is the ratio of area of the new square to the area of the original square?

9. On a Canadian map, the scale is 1: 25 million. If the distance between Toronto and Montreal is 2.1 cm on a map, what is the approximate distance between these cities?

10. When Jones leaves for Europe, the rate of exchange between the Canadian Dollars ($) and the EURO (€) is 1.00:0.69.
    a. How many Euros can Jones buy with $1200?
    b. How many Canadian dollars is needed to buy €2070?
    c. Upon his return, Jones noticed that the exchange rate has changed to 1.00:0.65. Now, he converted back €600 into Canadian dollars. How much money has he gained during his trip?

Percent: A rate expressed as part of 100. For example, ten of twenty-five students in Barry's class have a cold. This is written as 10 out of 25 or 10:25 or $\frac{10}{25}$ or $\frac{40}{100}$ or 40%.

Percent Increase: A rate used to express the change in one quantity as part of 100. For example, an increase of $0.10 on a price of $2.00 is an increase by fraction of $\frac{0.10}{2.00} = \frac{10}{200} = \frac{5}{100} = 5\%$, which can be expressed as an increase of 5%.

Simple Interest Formula: $I = Prt$
- I is the interest in dollars.
- P is the principal, that is the money borrowed, invested, or deposited.
- r is the annual interest rate as a decimal or as a percentage.
- t is the time in years.

11. A ball falls, rebounds to 72% of its previous height, and falls again. After two bounces, the ball reached a height of 25 cm. From what height was the ball drop originally?

12. Joanna borrowed $15000 at 3.0% simple interest per year for three years. Joanna pays back the money in equal monthly payments.

a. How much interest did Joanna owe?
b. How large is each monthly payment?

13. Mark lends $3350 to Ivan at a 6.25% per year. How much will Mark receive if Ivan repays the entire loan and interest after 8 months?

14. A family home was sold for $250 000. A commission of 6% was paid.
    a. How much was the commission?
    b. After paying the commission, how much do the owners receive for the house?
    c. How much would have been saved if the commission was only 3.5%?

15. Ben buys some apples at a wholesale store. The regular price of each apple is $1.25. However, the price of each apple would be $0.95 if Ben bought more than 20 apples.
    a. What is the cost of 20 apples?
    b. What is the cost of 30 apples?
    c. What is the price discount expressed as a percent?

FACT

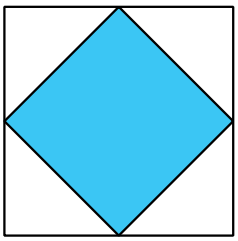
Interest is the "rent" paid for borrowing money, or the "rent" received for lending out money.

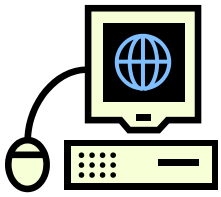
Commission is a service charge paid to an agent who arranges a purchase or a sale of property.

CHALLENGE YOURSELF!

16. The length of a rectangle is increased by 10% and the width is decreased by 20%. What percent is the new area compared with the original area?

17. The shaded square fits exactly into the big square as shown in the picture.
    a. Write a ratio to compare the side lengths of the two squares.
    b. Write a ratio to compare the area of the two squares.





Try These!
Determine the Ratio
http://www.321know.com/rat62bx2.htm
Unit Rates
http://www.321know.com/rat-unit-rate.htm

EXTENSIONS

Absolute Error is the positive difference between an exact value and an approximate value. For example, if an exact value is 10 and an approximate value is 9.9, then the absolute error is 0.1. Similarly, if the exact value is 10 and approximate value is 10.1, the absolute error is also 0.1. That is, the absolute error is the *positive* difference between the two numbers.

Percent Error is the absolute error divided by the exact value. The error can be seen as a percent change of the exact number. For example, if the exact value is 10 and the approximate value is 9.9, then the percent error is $\frac{10-9.9}{10}=\frac{0.1}{10}=\frac{1}{100}=1\%$.

General formula of Percent Error: $Percent\ Error = \frac{absolute\ error}{exact\ value} \times 100\%$

18. Determine the absolute difference between each pair of values.
    a. 9 and 10
    b. 90 and 100
    c. 10 and 1
    d. 100 and 91

19. Determine the percent error. (Use the second number as the exact value.)
    a. 9 and 10
    b. 90 and 100
    c. 1 and 10
    d. 91 and 100

20. There are 1500 students in a school. You guess that there are 1400 students.
    a. By how many students is your guess off?
    b. Calculate the percent error of your guess?

21. Each year, a city on average has 15 days above 30 degrees Celsius. This year there were only 12 days above 30 degrees Celsius.
    a. By how many days were your observations off?
    b. Calculate the percent error of your observations?

22. A road map states that it is 976 km to the capital city. While travelling by car, you measure the distance to be 985 km.
    a. By how many kilometres is your measurement off?
    b. What was the percent error of your measurement?