

Grade 9

Geometry: Polygons

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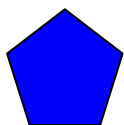
Play the **Interior Angles Matching Game** <http://www.studystack.com/matching-88926>

You may also go to www.wiredmath.ca for the link.

A **polygon** is a shape enclosed by a path of non-intersecting sides that end at the starting point.

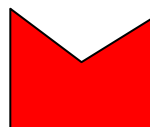
A **convex** polygon has no interior angle greater than 180° .

E.g.

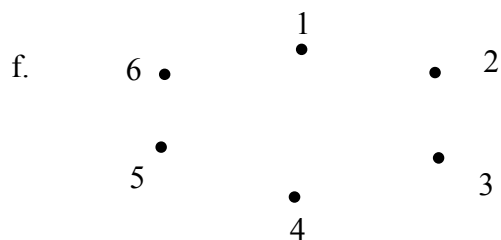
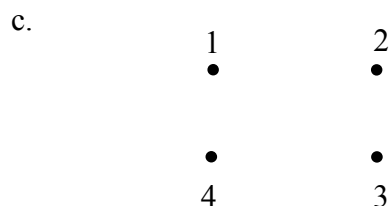
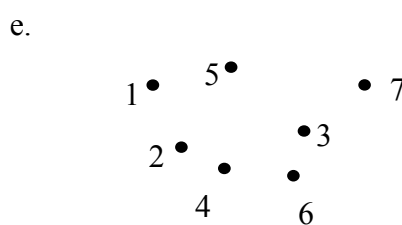
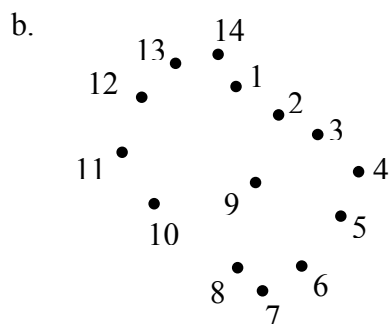
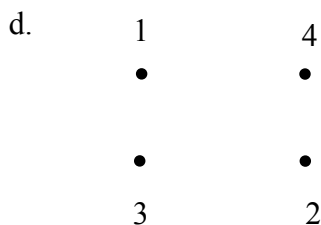
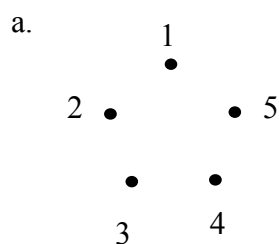


A **concave** polygon has at least one angle greater than 180° .

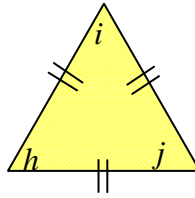
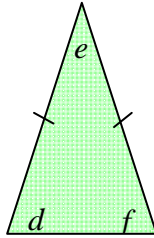
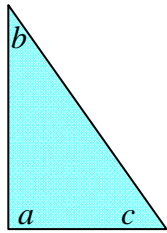
E.g.



1. Connect the dots in order. Always connect back to 1. Determine if the dots produce a polygon, and if so, label it as convex or concave.



Triangle Man's Lesson:

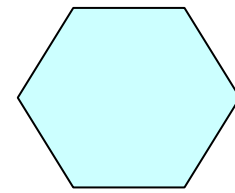
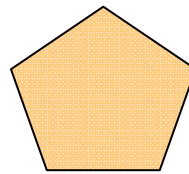
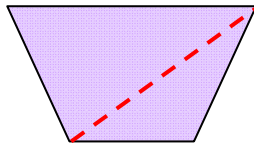
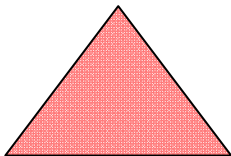


$$a + b + c = d + e + f = h + i + j = 180^\circ$$

Remember kids, be it a scalene, isosceles or equilateral triangle, the sum of the interior angles of a triangle will always be 180° !



2. a. From one vertex, draw diagonals to connect non-adjacent vertices, if possible.



- b. What shapes are formed within the polygons by drawing these diagonals?
c. Complete the following chart.

Number of sides	3	4	5	6	100	n
Number of diagonals drawn from only one vertex	0	1				
Number of Triangles	1	2				
Total sum of the interior angles of the triangles	180°	360°				

- d. Derive a formula to calculate the sum of the interior angles of a polygon with n sides.

Sum of Interior Angles Theorem

A convex polygon with n sides has a total sum of its interior angles equal to:

3. Calculate the total sum of the interior angles for the following convex polygons.
- | | |
|------------|---------------|
| a. 4 sides | c. 9 sides |
| b. 6 sides | d. 1000 sides |
4. Determine the number of sides of each convex polygon with the following interior angle sums.
- | | |
|----------------|---------------------|
| a. 540° | c. 1080° |
| b. 900° | d. $170\ 100^\circ$ |

Naming Polygons

The number of sides categorizes polygons. We often talk about triangles, quadrilaterals and pentagons (with prefixes of “tri”, “quad” and “penta” meaning 3, 4 in Latin, and 5 in Greek) but have you ever heard of a nonagon (9 sides), a hendecagon (11 sides) or even a dodecagon (12 sides)? Do not worry; we often name a polygon with n sides an n -gon for simplicity. For example, an octagon is also an 8-gon.

5. A regular polygon has all angles equal. Calculate the degree measure of each interior angle.
- | | |
|-----------------------|-------------------------------------|
| a. a square | c. a regular heptagon (seven sides) |
| b. a regular pentagon | d. a regular 99-gon |

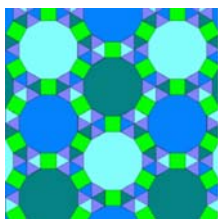
6. A regular soccer ball is composed of 32 faces. 12 of these are pentagons and 20 of them are hexagons (the shape is called a truncated icosahedron). What is the total sum of the interior angles of each shape on the soccer ball?



7. In a bowl of corn chips, 80% of the chips are isosceles triangles, while the remaining 20% of the chips are broken into chip fragments, a triangle portion and a trapezoid portion. If the sum of the interior angles of all the chips and chip fragments is $12\ 600^\circ$, how many chips were there in the bowl?

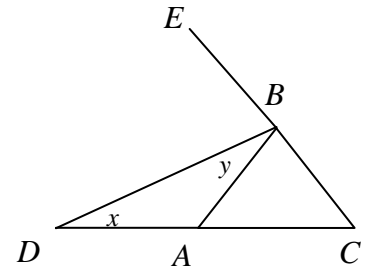


Check This Out!



Dodecagons (12-gon), squares (4-gon) and triangles (3-gon) coming together to make a nice tiling. Isn't math beautiful? Try making your own tiling at home.

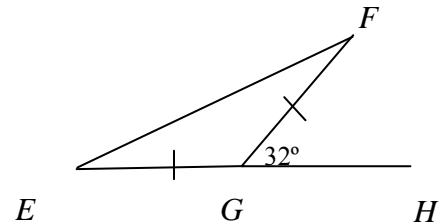
8. In $\triangle ABC$, CA is extended to D and CB is extended to E .
- If $\angle ABC$ is 47° and $\angle ACB$ is 49° , what is the measure of $\angle DAB$?
 - Determine $\angle ABE$.
 - If $\angle ABC$ is b and $\angle ACB$ is c , write a formula for $\angle DAB$ in terms of b and c .
 - If we join D and B , we form a new triangle, $\triangle BCD$.
If $\angle ABD$ is y and $\angle ADB$ is x , what is the measure of $\angle BAC$ in terms of x and y ?
 - Can $\angle ABD$ ever be greater than $\angle BAC$? Why or why not?



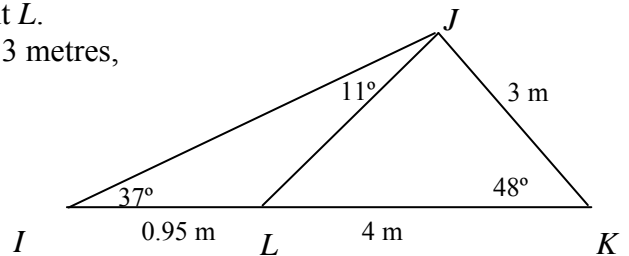
Exterior Angle of a Triangle

In any triangle, if one side is extended, then the exterior angle formed will be equal to:

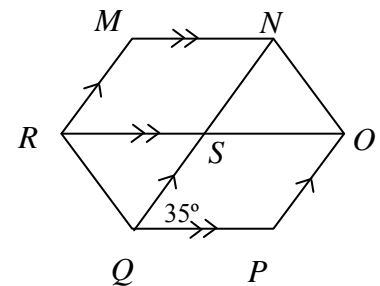
9. In isosceles $\triangle EFG$, $GE = GF$. EG is extended to point H .
If $\angle HGF$ is 32° , what is the measurement angle of $\angle GEF$?



10. In $\triangle IJK$, a line is drawn from point J to meet IK at point L .
If $\angle LIJ$ is 37° , $\angle IJL$ is 11° , $\angle JKI$ is 48° , and JK is 3 metres, how long is JL ?



11. In hexagon $MNOPQR$, diagonals NQ and RO are drawn and meet at point S to create two parallelograms as well as two triangles.
If $\angle NQP$ is 35° , what is the sum of $\angle ORQ$ and $\angle NQR$?



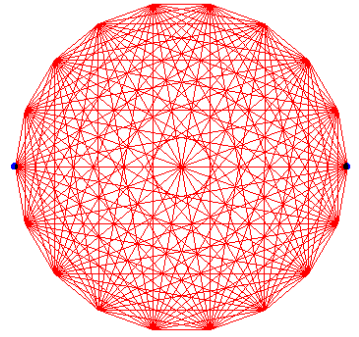
Did You Know?

Euclid was a very famous mathematician who wrote the book *Elements*, which contains many of the geometric theorems we use today. In addition to being known as the father of geometry, he also proved that there are **infinitely** many primes!

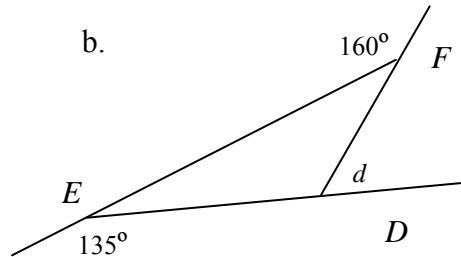
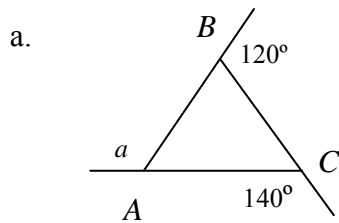


EXTENSIONS

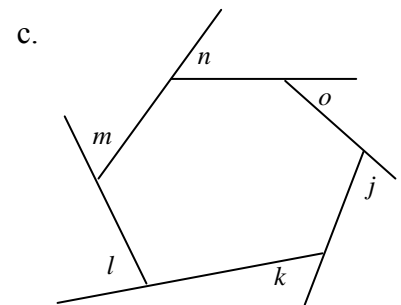
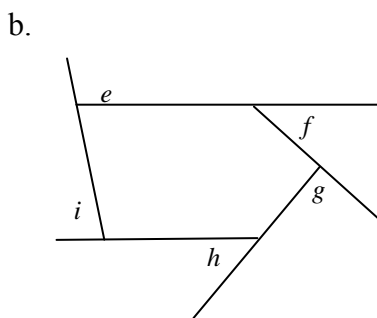
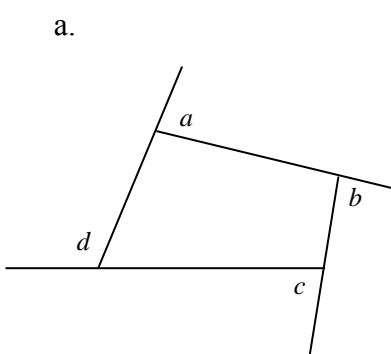
12. In the diagram the polygon has 18 sides. From each vertex, diagonals are drawn to connect to all non-adjacent vertices. How many diagonals are there? Derive a formula for the number of diagonals in any polygon with n sides.



13. Calculate the sum of the exterior angles of the following triangles.



14. Calculate the sum of the exterior angles for each figure.

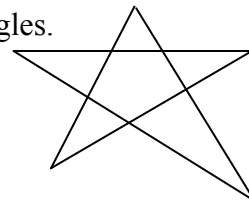


d. Did you find a pattern? Show this is true with an n -gon.

Sum of Exterior Angles of a Polygon

The sum of the measures of the exterior angles of a polygon, one at each vertex equals

15. The figure to the right is a star created out of a pentagon and 5 triangles. Determine the sum of the angles of the points of the star.



Don't forget to try these math drills now! Go to www.wiredmath.ca for the link.

TRY THIS!

Polygons and Angles

<http://www.quia.com/pop/176616.html>

