Grade 7
PATTERNING AND ALGEBRA - MODELLING

Answers:

1. i. a. 
   \[
   \begin{array}{ccccccc}
   & 3 & 6 & 9 & 12 & 15 & 18 & 21 \\
   \end{array}
   \]
   b. Starting with three, we add three to get the next term. Alternatively, each term is
   three times the term number.
   c. 10\textsuperscript{th} term: 30
   50\textsuperscript{th} term: 150
   \(n\textsuperscript{th}\) term: 3\(n\)
   
   ii. a. 
   \[
   \begin{array}{ccccccc}
   & -3 & 1 & 5 & 9 & 13 & 17 & 21 \\
   \end{array}
   \]
   b. Starting with negative three, we add four to get the next term. Alternatively, each
   term is four times the term number minus seven.
   c. 10\textsuperscript{th} term: 33
   25\textsuperscript{th} term: 93
   \(n\textsuperscript{th}\) term: 4\(n - 7\)
   
   iii. a. 
   \[
   \begin{array}{ccccccc}
   & \frac{1}{2} & 1 & 2 & 4 & 8 & 16 & 32 \\
   \end{array}
   \]
   b. Starting with one half, we multiply by two to get the next term. Alternatively, each
   term is two raised to the exponent (the term number minus 2).
   c. 10\textsuperscript{th} term: 256
   100\textsuperscript{th} term: 2\(^98\)

2. a. 
   \[
   \begin{array}{ccccccc}
   x & 0 & 1 & 2 & 3 & 4 \\
   y & 0 & 2 & 4 & 6 & 8 \\
   \end{array}
   \]
   Description: Starting with zero, we add two to get each successive term.

   b. 
   \[
   \begin{array}{ccccccc}
   x & 0 & 1 & 2 & 3 & 4 \\
   y & 5 & 4 & 3 & 2 & 1 \\
   \end{array}
   \]
   Description: Starting with five, we subtract one to get each successive term.

   c. 
   \[
   \begin{array}{ccccccc}
   x & 0 & 1 & 2 & 3 & 4 \\
   y & 20 & 17 & 14 & 11 & 8 \\
   \end{array}
   \]
   Description: Starting with twenty, we subtract three to get each successive term.

   d. 
   \[
   \begin{array}{ccccccc}
   x & 0 & 1 & 2 & 3 & 4 \\
   y & -3 & 4 & 11 & 18 & 25 \\
   \end{array}
   \]
   Description: Starting with negative three, we add seven to get each successive term.

   The coefficient of \(x\) is the number added to get each successive term, and the constant is
   the first term.

3. a. Starting with zero (at \(x = 0\)), we subtract one to get each successive term.

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b. Starting with six (at \( x = 0 \)), we add four to get each successive term.

c. Starting with negative one (at \( x = 0 \)), we add three to get each successive term.

4. If we group \( 50 - 49 + 48 - 47 + 46 - 45 + \ldots + 4 - 3 + 2 - 1 \) as:
\[
(50 - 49) + (48 - 47) + (46 - 45) + \ldots + (4 - 3) + (2 - 1),
\]
we get
\[
1 + 1 + 1 + \ldots + 1 + 1.
\]
Since there are 25 ones in this sum, the answer is 25.

5. Looking at the fours in the decimal, they appear in positions 3, 6, 9, \ldots. Therefore, in every position that is a multiple of three, there is a four. Since 51 is a multiple of three, there is a four in position 51. Therefore, there is a three in position 50.

6. In the pattern, we increase by seven with each successive term. Since eight is one larger than seven, the pattern consists of all numbers that are one larger than multiples of seven. The only multiple of seven that is between 150 and 160 is 154. Therefore, the number that appears in the pattern that is between 150 and 160 is 155.

7. Since one sequence increases by four each time and the other increases by five each time, the two will match up every twenty (four times five). Therefore, if they match up at thirteen, they will again match up at \( 13 + 20 = 33 \).

8. Let the number of raisins that Lily ate on the third day be \( x \). Then on the second day, she ate \( x - 7 \), on the first day she ate \( x - 14 \), on the fourth day she ate \( x + 7 \), and on the fifth day she ate \( x + 14 \). Adding the five days and setting equal to the total number of raisins, we get:
\[
(x - 14) + (x - 7) + x + (x + 7) + (x + 14) = 150
\]
\[
5x = 150
\]
\[
x = 30
\]
a. Therefore, she ate 30 raisins on the third day.
b. On the first day, she ate \( 30 - 2(7) = 16 \) raisins. Alternatively, on the first day she ate \( x - 14 \) raisins, and \( x = 30 \) so she ate \( 30 - 14 = 16 \) raisins.
9. It will take nine days for the plant to reach half of its maximum height. If it doubles in height each day, and if it was at half its maximum height on day nine, it would double \( 2 \times \frac{1}{2} = 1 \) to its full maximum height in one day, on day ten.

10. a. First, we want to find the largest number of marbles we can place so that there are an equal number of each colour. Since the most we can have of either colour (if the two have equal amounts) is 50, we are looking for the largest sum of \( 1 + 2 + 3 + \ldots \) that is less than 50. We can see that \( 1 + 2 + 3 + \ldots + 8 + 9 = 45 \). That means that after placing nine green and nine blue marbles, we have \( 45 + 45 = 90 \) marbles placed. We then place 10 green marbles, and that takes us up to 100 marbles. Therefore, the 100th marble placed is green.

b. We know from part a) that of the first 90 marbles, 45 are green. We also know that the last 10 are green. This gives a total of 55 green marbles.